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Mitigation of vortex-induced vibration in bridge structure using nonlinear energy sink inerter: Weight effect and compensational strategies

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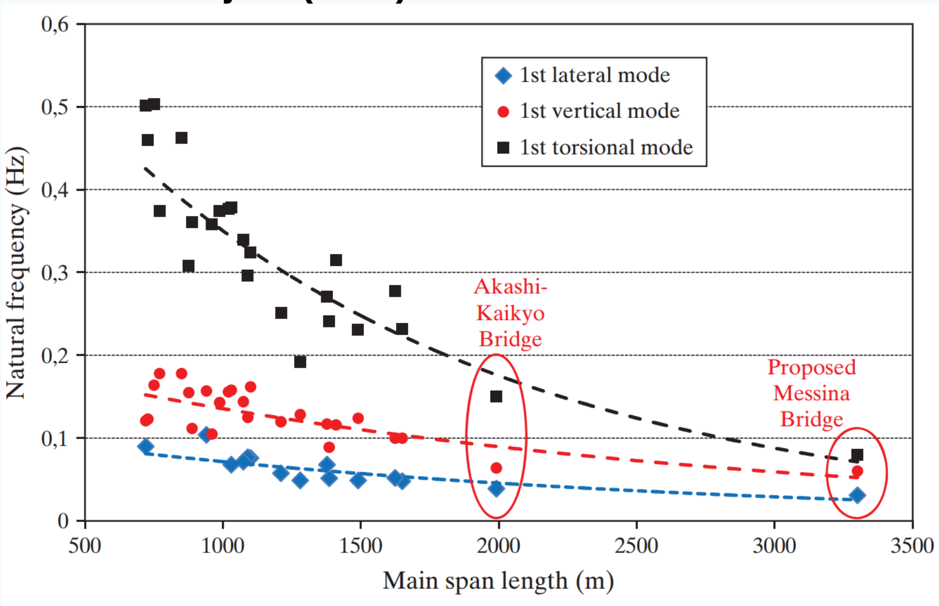
Contents

1. Background

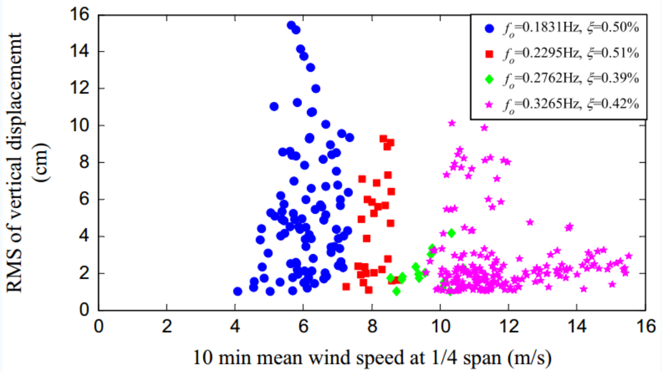
Significant wind sensitivity of bridges

Yozo Fujino (2013)

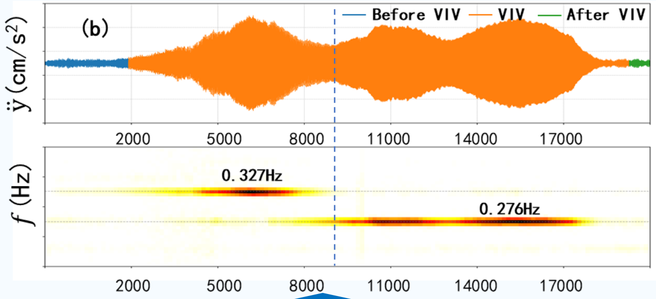


Frequency evolution with span length

Li et al. (2014)



Frequent vibrations



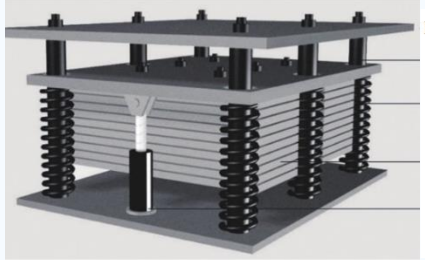
Multimodal vibration (Xihoumen)

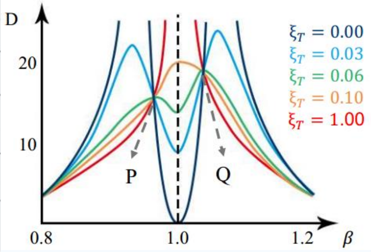
Low frequency and multimodal vortex-induced vibration !

1. Background

Why is nonlinear energy sink inerter (NESI)?

Linear dynamic vibration absorber



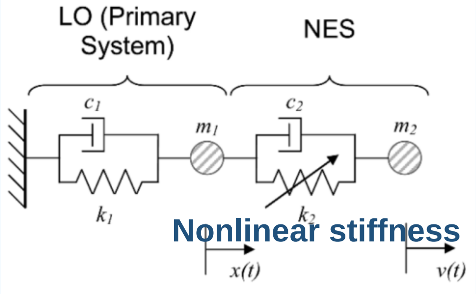


Tuned mass damper (TMD) **Narrow bandwidth**

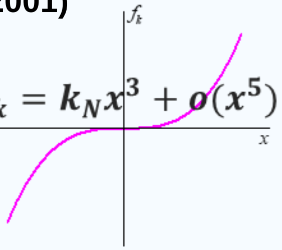
Large static stroke: $\Delta \approx 0.25/f^2$

Frequency (Hz)	Static stroke (m)
$f = 0.20$	6.25
$f = 0.15$	11.11
$f = 0.10$	25.00

Nonlinear dynamic vibration absorber



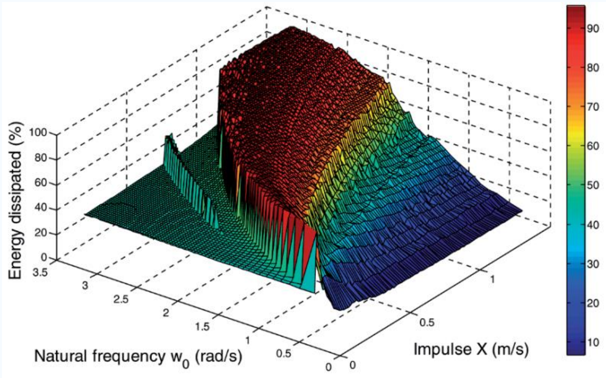
Gendelman et al. (2001)



$$f_k = k_N x^3 + o(x^5)$$

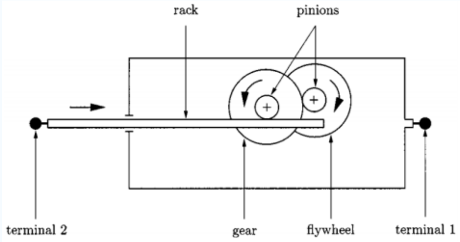
Nonlinear nergy sink (NES)

Vakakis et al. (2008)



Frequency bandwidth expanded by NES

Inerter enhanced vibration absorber

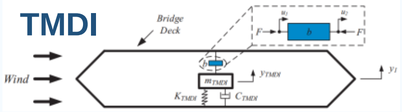


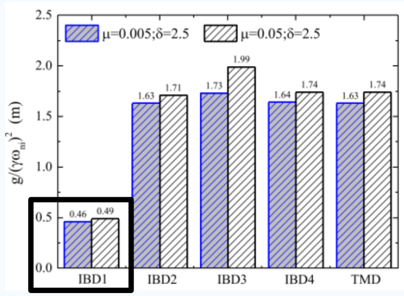
Inertance

$$f_I = b(\ddot{y}_1 - \ddot{y}_2)$$

Inerter proposed by Smith. (2002)

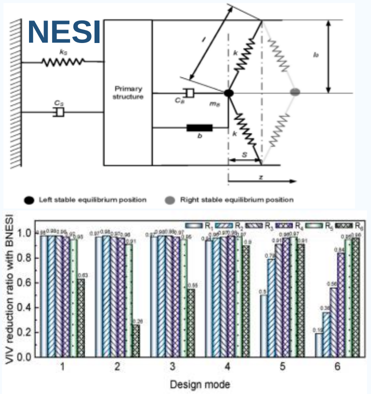
Xu et al. (2019; 2020)





Small static stroke

Xie et al. (2024)

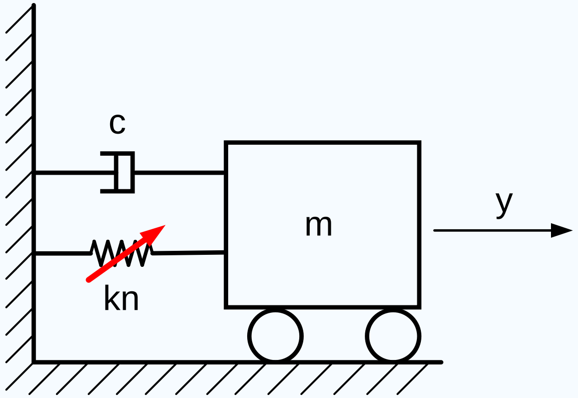


Multimodal vibration mitigation

NESI is capable of small static stroke and multimodal vibration mitigation !

1. Background

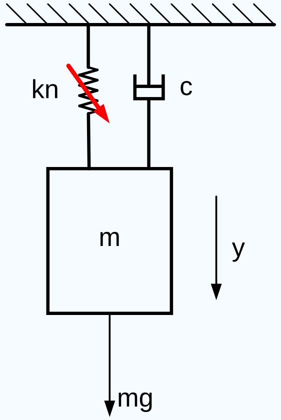
Non-self-compensating nature of NESI weight



$$m\ddot{y} + k_n y^3 + c\dot{y} = 0$$

Equilibrium equation

NES under lateral vibration



$$k_n \Delta^3 = mg$$

$$m\ddot{y} + k_n (y + \Delta)^3 + c\dot{y} = mg$$

Weight effct terms

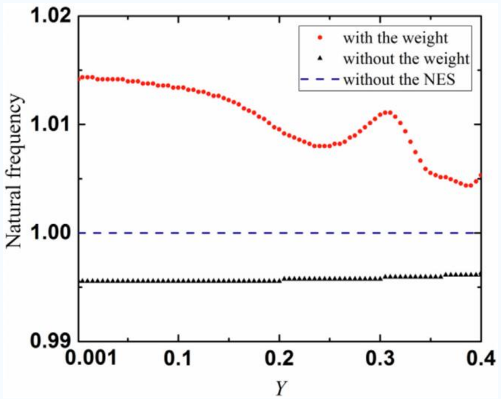
$$m\ddot{y} + k_n (y^3 + 3y^2\Delta + 3y\Delta^2) + c\dot{y} = 0$$

Equilibrium equation

NES under vertical vibration

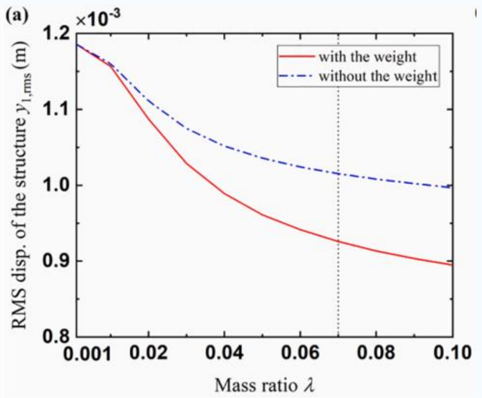
Forced vibration under weight

Chen et al. (2019)



Stochastic excitations under weight

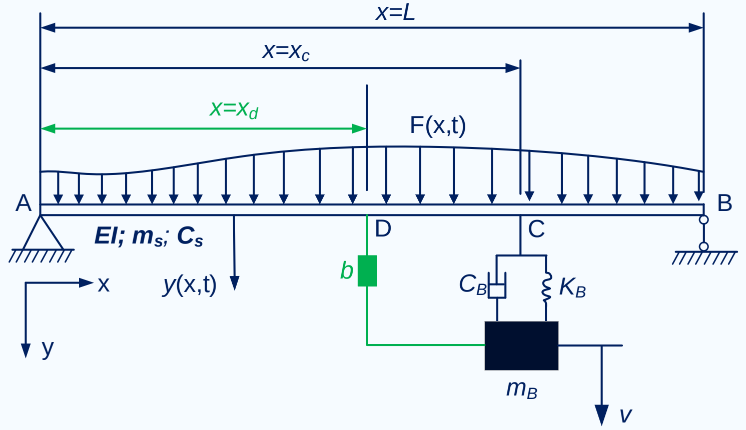
Li et al. (2022)



- **Variation in vibration frequency**
- **Smaller vibration amplitude**
- **How about VIV mitigation？**

2. Modelling of systems

NESI-beam coupled system considered weight



- Primary structure: **Euler-Bernoulli beam**
- Auxiliary structure: **Bistable NESI** and **Cubic NESI**
- External load: **Scanlan** nonlinear vortex-excited force model
- Inertance coefficient: $\alpha = 0.01$ (100x mass amplification)

Dynamic balance system

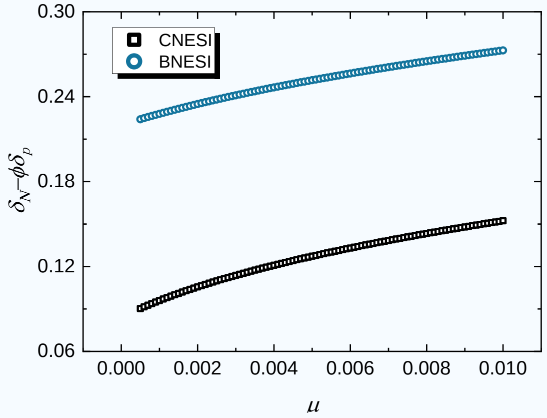
$$\left(A_p(\tau) + \boldsymbol{\delta_P}\right) + 2\xi_p \dot{A}_p(\tau) + \ddot{A}_p(\tau)$$
$$+ \left\{k_{n1}\left[V(\tau) + \boldsymbol{\delta_B} - \phi_p(x_c)\left(A_p(\tau) + \boldsymbol{\delta_P}\right) - s\right] - k_{n2}\left[V(\tau) + \boldsymbol{\delta_B} - \phi_p(x_c)\left(A_p(\tau) + \boldsymbol{\delta_P}\right) - s\right]^3 - 2\lambda\left[\dot{V}(\tau) - \phi_p(x_c)\dot{A}_p(\tau)\right]\right\}\phi_p(x_c)$$
$$- \beta\left[\ddot{V}(\tau) - \phi_p(x_d)\ddot{A}_p(\tau)\right]\phi_p(x_d) = Q(\tau)$$

$$\mu\ddot{V}(\tau) - k_{n1}\left[V(\tau) + \boldsymbol{\delta_B} - \phi_p(x_c)\left(A_p(\tau) + \boldsymbol{\delta_P}\right) - s\right] + k_{n2}\left[V(\tau) + \boldsymbol{\delta_B} - \phi_p(x_c)\left(A_p(\tau) + \boldsymbol{\delta_P}\right) - s\right]^3 + 2\lambda\left[\dot{V}(\tau) - \phi_p(x_c)\dot{A}_p(\tau)\right]$$
$$+ \beta\left[\ddot{V}(\tau) - \phi_p(x_d)\ddot{A}_p(\tau)\right] = \boldsymbol{(\mu + \alpha\beta)\gamma}$$

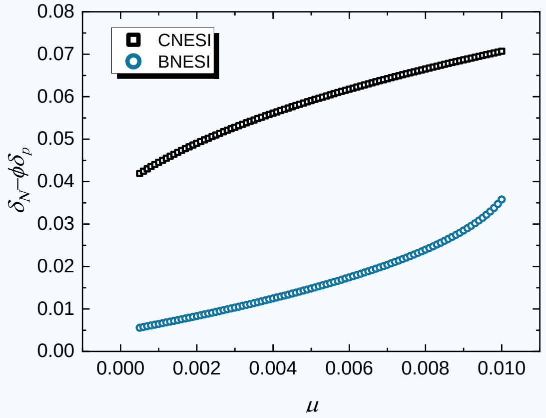
$$Q(\tau) = \frac{U_r}{2\pi m_r}\int_0^1 \left[Y_1\left(1 - \varepsilon\phi_p^2(lL)A_p^2(\tau)\right)\right]\phi_p^2(lL)\dot{A}_p(\tau)dl$$

2. Modelling of systems

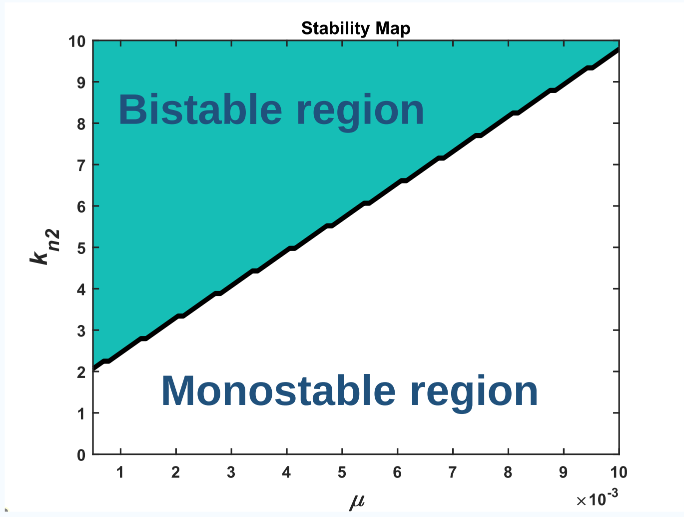
Static equilibrium characteristics



(a) $k_{n2} = 1$



(b) $k_{n2} = 10$



Static stroke of NESI with different mass ratios

Stability map of BNESI

- The static stroke of NESI is significantly influenced by nonlinear stiffness k_{n2}
- The stability characteristics of BNESI are also influenced by non-linear stiffness k_{n2}

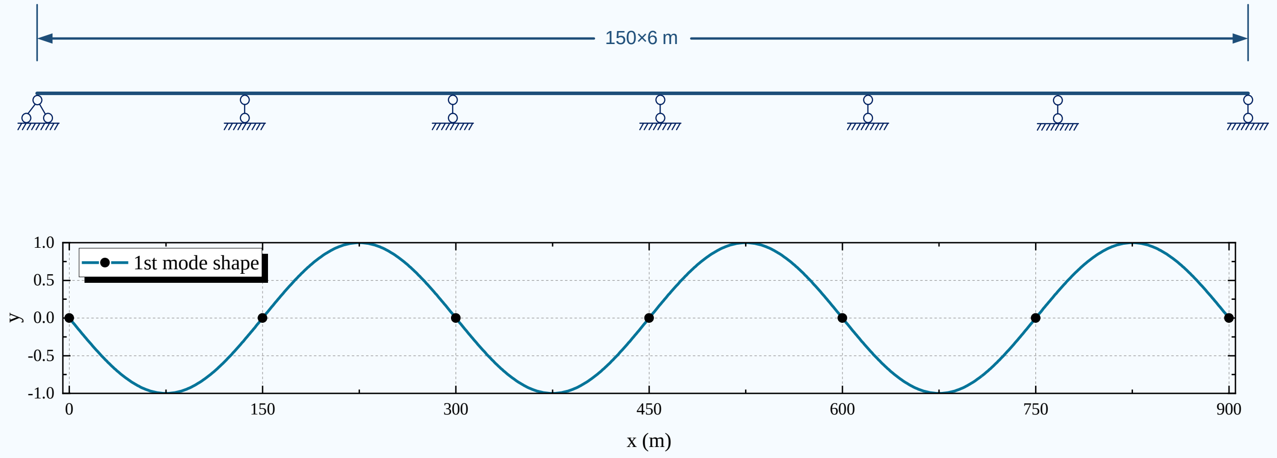
Reference bridge and VIV characteristics

Yu et al. (2022)

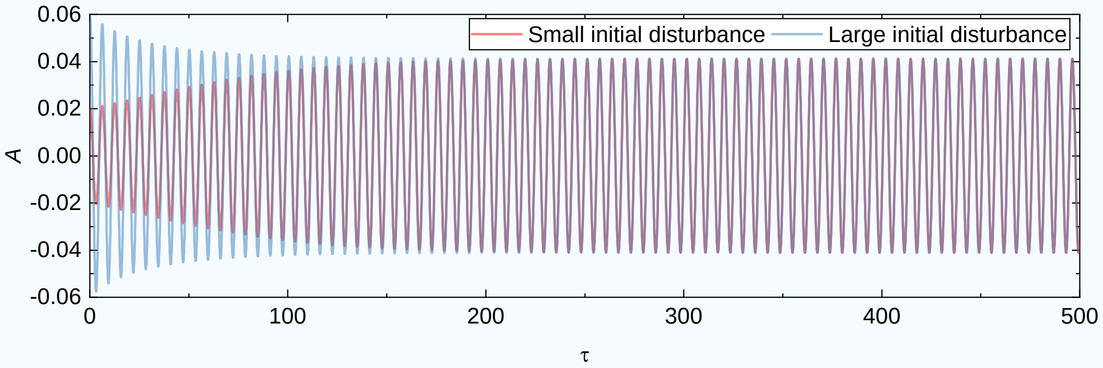
- **Reference bridge:** $m_s = 19800kg/m$, $EI = 7.6 \times 10^{11} N \cdot m^2$, $D = 4.5m$, $\xi = 0.1\%$, $m_p = 8887.5t$, $f_p = 0.433Hz$

Dai et al. (2021)

- **VIV aerodynamic parameters:** $U_r = 8.14$, $\rho = 1.225kg/m^3$, $Y_1 = 15.7$, $\varepsilon = 2900$



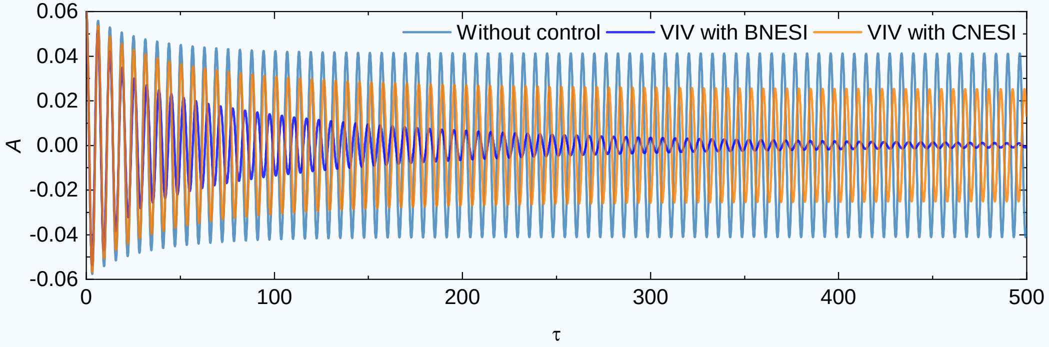
Span arrangement and basic modal shape



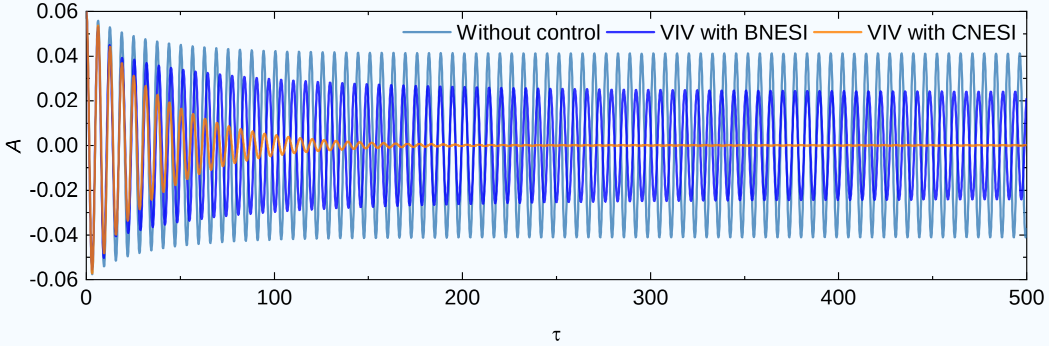
VIV evolution of reference bridge

3. Weight effect on VIV mitigation

VIV mitigation with and without self-excited characteristic

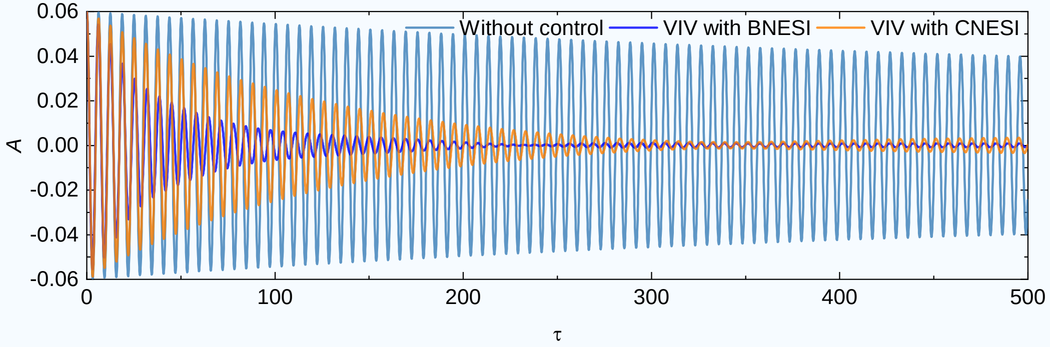


(a) Without weight

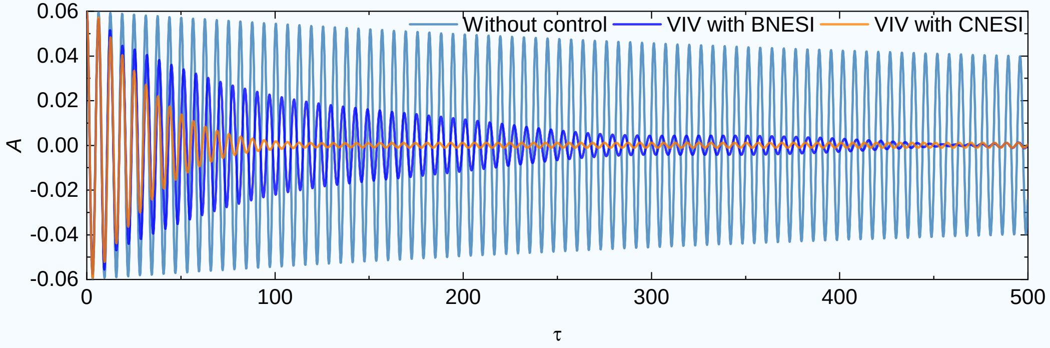


(b) With weight

Displacement of beam structure under VIV



(a) Without weight

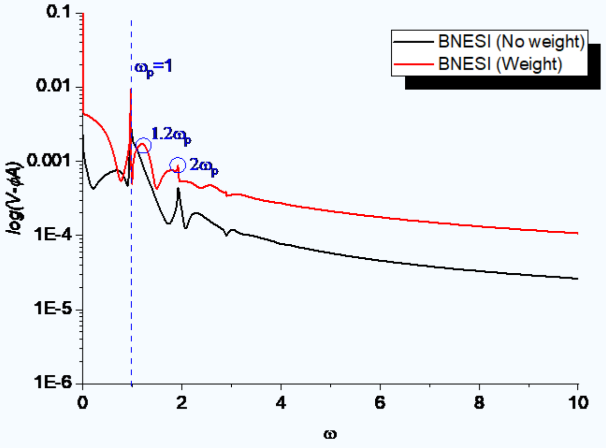


(b) With weight

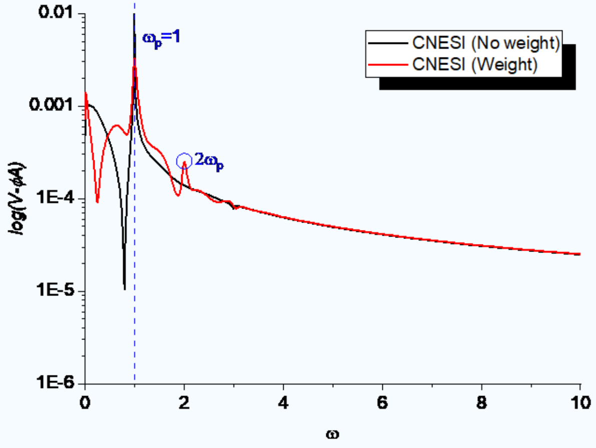
Displacement of beam structure under HFV

- Weight effect leads to a degradation in the control performance of BNESI, while it improves that of CNESI
- Mitigating VIV with self-exciting characteristics is more challenging than mitigating HFV

VIV mitigation with and without self-excited characteristic

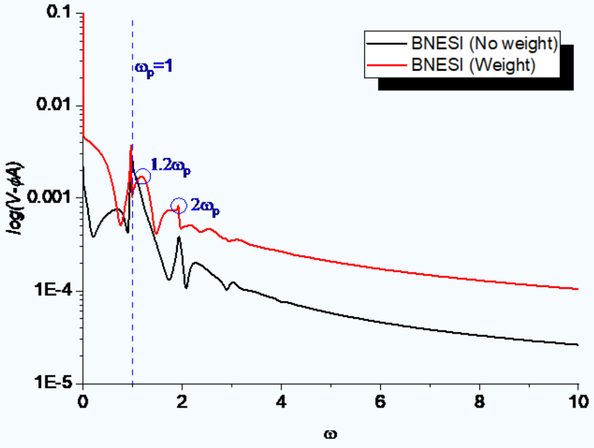


(a) BNESI

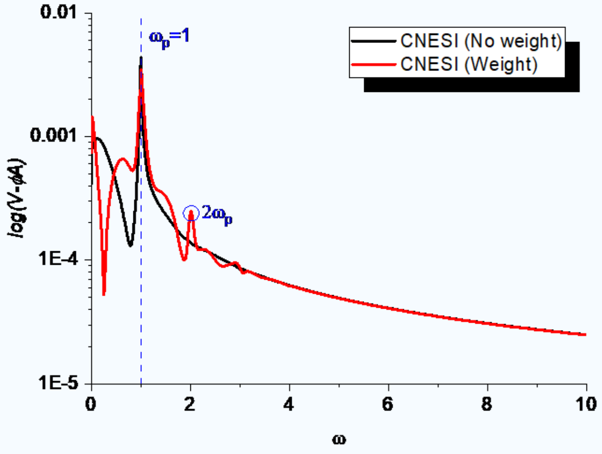


(b) CNESI

Amplitude spectra of NESI under VIV



(a) BNESI



(b) CNESI

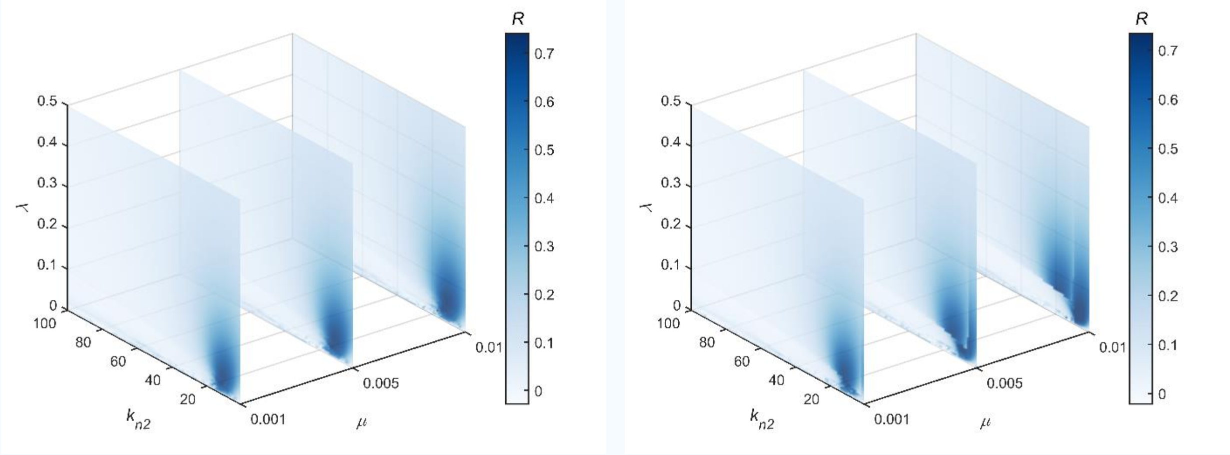
Amplitude spectra of NESI under HFV

- Weight effect leads to additional transient resonance capture (TRC) frequency ratios
- Integer multiple frequency ratios facilitate vibration mitigation, whereas non-integer multiple frequency ratios hinder it

3. Weight effect on VIV mitigation

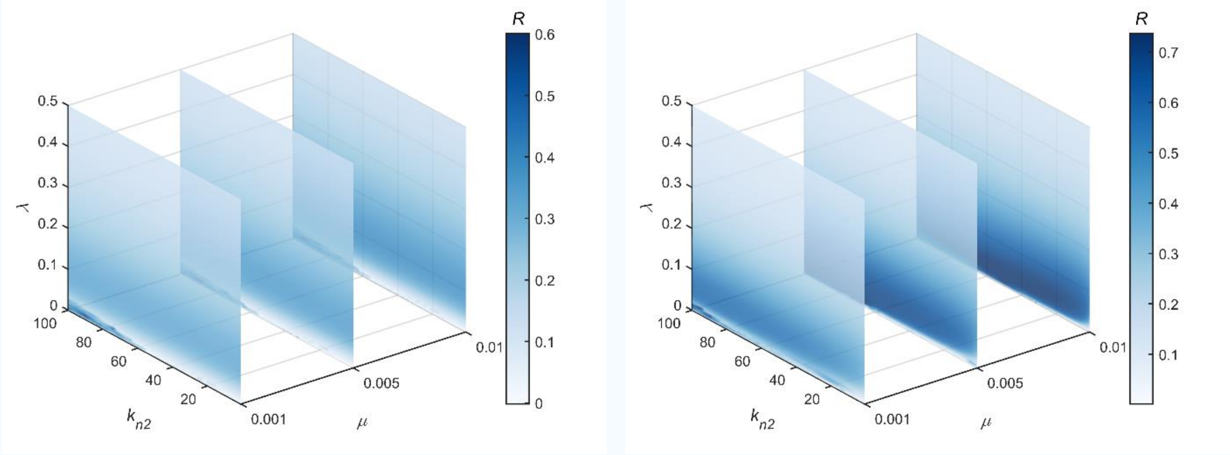
Effective parameter distribution for VIV mitigation

VIV reduction ratio: $R = \frac{RMS_u - RMS_c}{RMS_u}$



(a) Without weight (b) With weight

VIV reduction ratio under BNESI



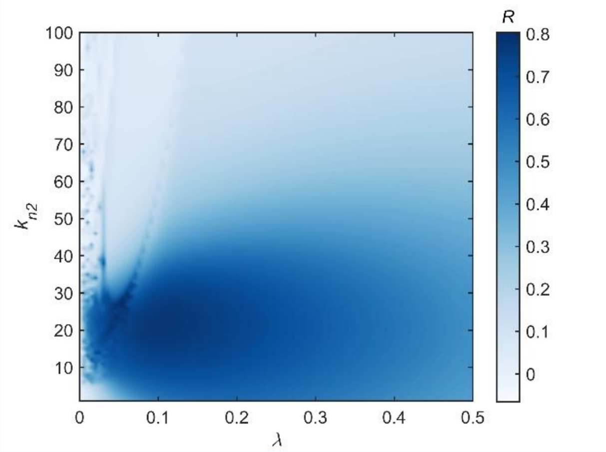
(a) Without weight (b) With weight

VIV reduction ratio under CNESI

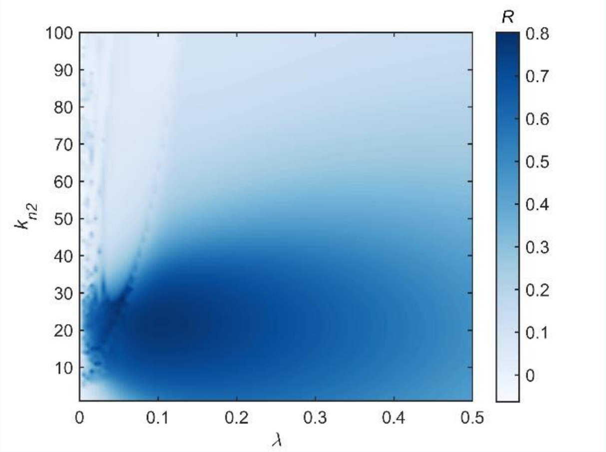
- **The effective parameter distribution range of NESI differs significantly with and without weight**
- **A larger mass ratio leads to a more significant weight effect and a wider range of effective parameter**

4. Compensational strategies

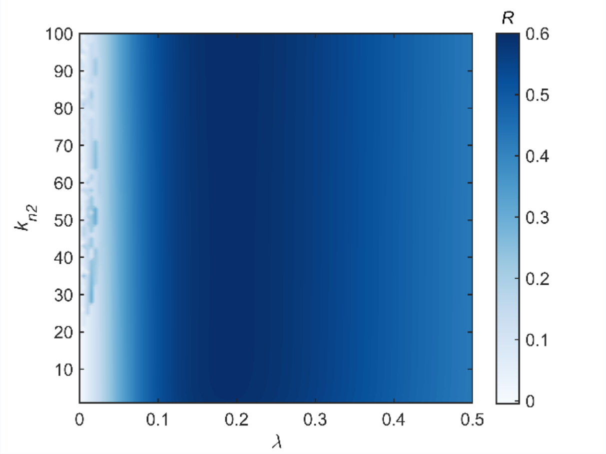
Small inertance coefficient



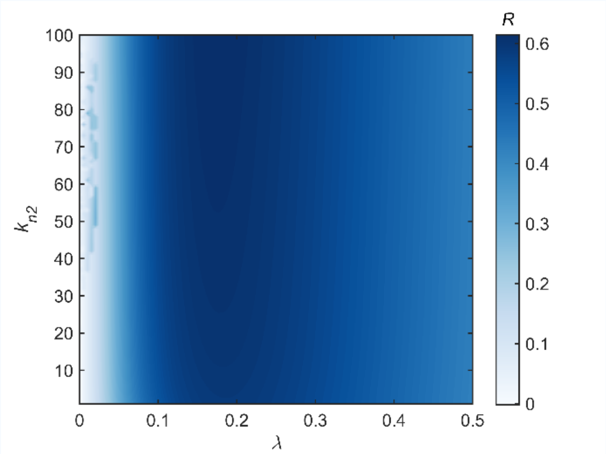
(a) BNESI without weight



(b) BNESI with weight

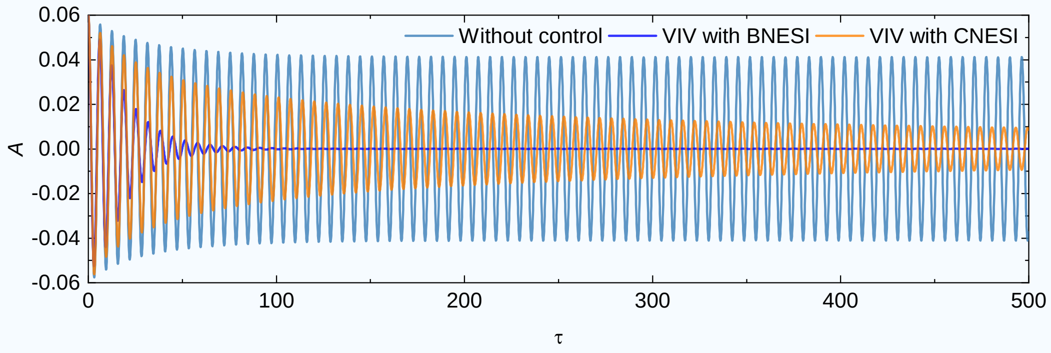


(c) CNESI without weight

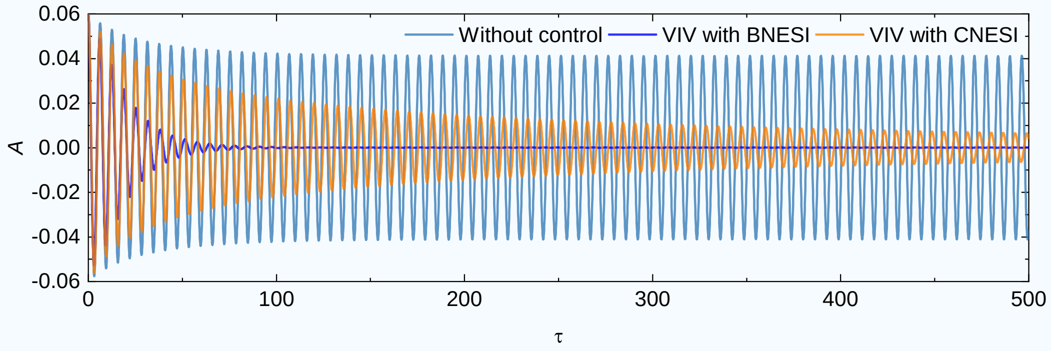


(d) CNESI with weight

VIV reduction ratio with α=1/5000



(a) Without weight

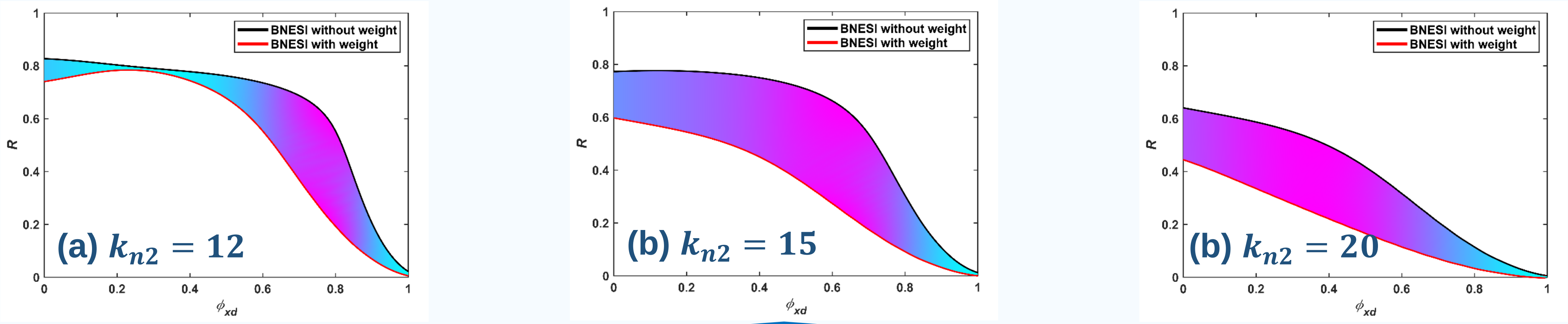


(b) With weight

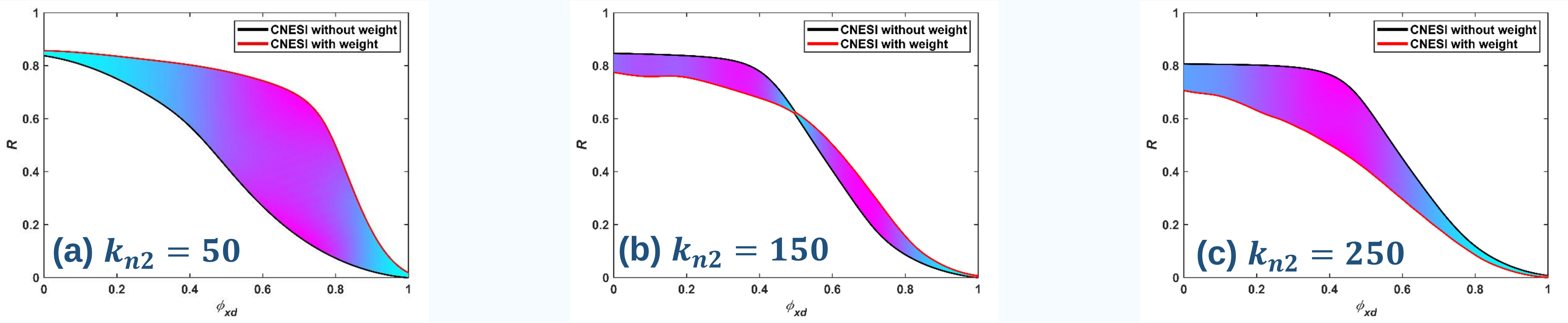
Displacement of beam with α=1/5000

4. Compensational strategies

Right modal difference of inerter

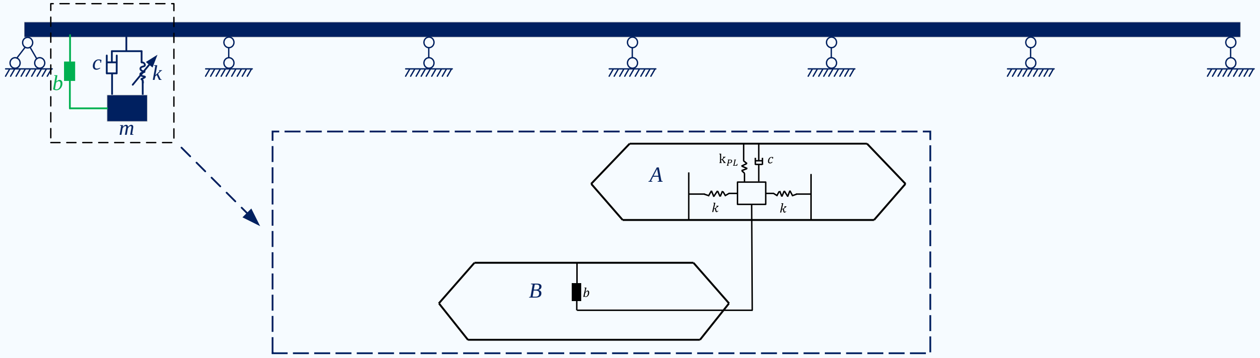


VIV reduction ratio under BNESI varied with ϕ_{xd}



VIV reduction ratio under CNESI varied with ϕ_{xd}

Supplementary weight-balanced stiffness element



PL-NESI system

- Stiffness system: **Positive linear stiffness element (k_{PL})** and **nonlinear stiffness element**
- Weight self-balanced: $k_{PL}\Delta = (m_N + m_I)g$

Dynamic balance system

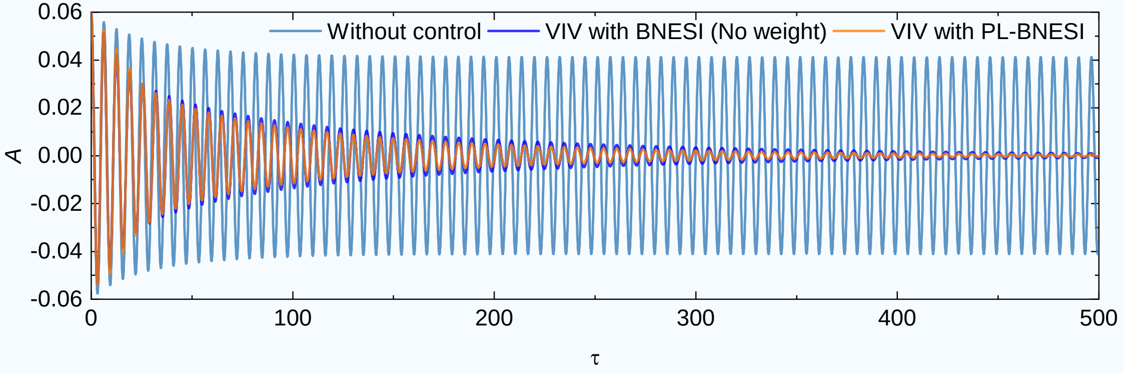
$$A_p(\tau) + 2\xi_p \dot{A}_p(\tau) + \ddot{A}_p(\tau) + \left\{-k_{PL}\left[V(\tau) - \phi_p(x_c)A_p(\tau)\right] + k_{n1}\left[V(\tau) - \phi_p(x_c)A_p(\tau) - s\right] - k_{n2}\left[V(\tau) - \phi_p(dx_c)A_p(\tau) - s\right]^3 - 2\lambda\left[\dot{V}(\tau) - \phi_p(x_c)\dot{A}_p(\tau)\right]\right\}\phi_p(x_c) - \beta\left[\ddot{V}(\tau) - \phi_p(x_d)\ddot{A}_p(\tau)\right]\phi_p(x_d) = Q(\tau)$$

$$\mu\ddot{V}(\tau) + k_{PL}\left[V(\tau) - \phi_p(x_c)A_p(\tau)\right] - k_{n1}\left[V(\tau) - \phi_p(x_c)A_p(\tau) - s\right] + k_{n2}\left[V(\tau) - \phi_p(x_c)A_p(\tau) - s\right]^3 + 2\lambda\left[\dot{V}(\tau) - \phi_p(x_c)\dot{A}_p(\tau)\right] + \beta\left[\ddot{V}(\tau) - \phi_p(x_d)\ddot{A}_p(\tau)\right] = 0$$

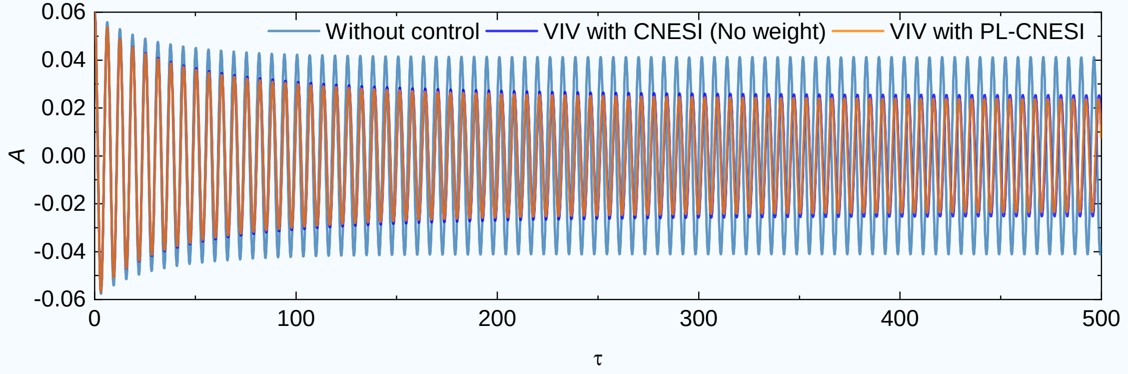
$$Q(\tau) = \frac{U_r}{2\pi m_r}\int_0^1 \left[Y_1\left(1 - \varepsilon\phi_p^2(lL)A_p^2(\tau)\right)\right]\phi_p^2(lL)\dot{A}_p(\tau)dl$$

4. Compensational strategies

Supplementary weight-balanced stiffness element



Displacement of beam with BNESI/PL-BNESI



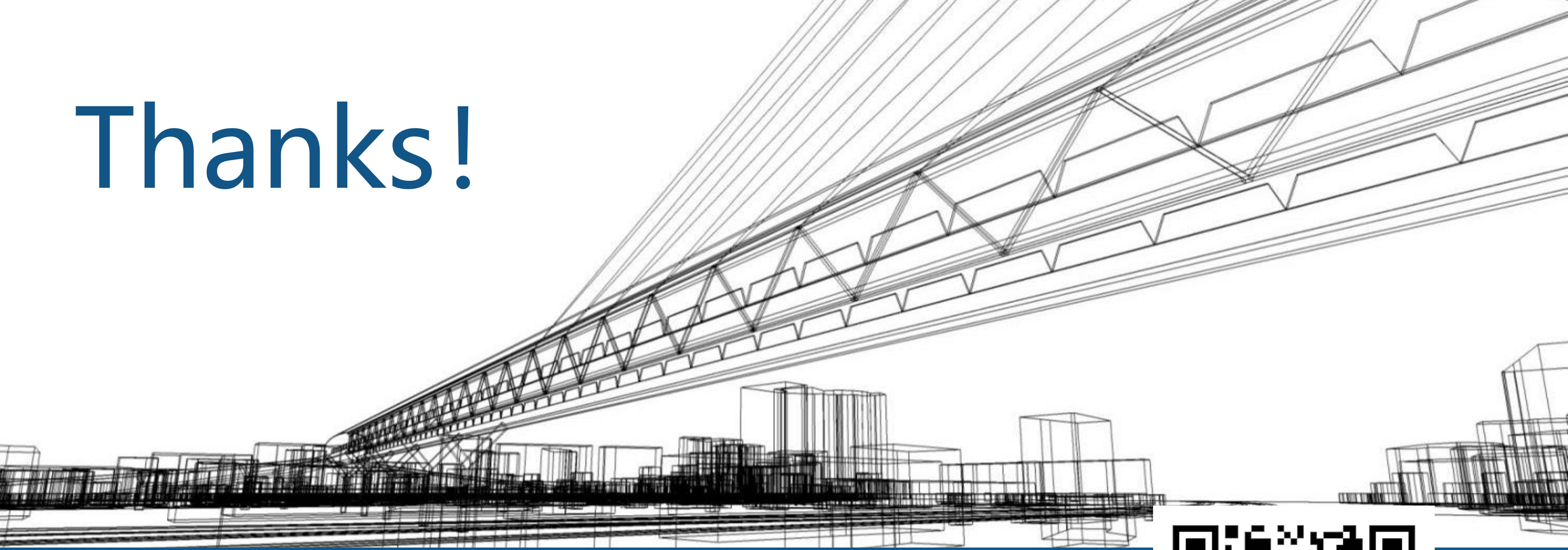
Displacement of beam with CNESI/PL-CNESI

- **Introducing a positive linear stiffness element can eliminate the weight effect and achieve control consistent with an ideal NESI**

5. Conclusions

- The weight effect in the NESI system significantly alters the effective parameter range, with integer multiple TRC frequency ratios enhancing TET efficiency, while non-multiple ratios reduce it
- VIV is more sensitive to weight effects, cannot be adequately represented by HFV
- Reducing the inertance coefficient and increasing the spacing at both ends can effectively mitigate weight effects on VIV mitigation, enhancing parameter robustness
- The PL-NESI system effectively balances weight with individual linear stiffness elements, isolating weight effects on nonlinear stiffness

Thanks!



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